Hindawi
Advances in Astronomy
Volume 2021, Article ID 6680069, 10 pages
https://doi.org/10.1155/2021/6680069

Research Article

Traversable Wormholes Existence in $f(R, T)$ Gravity Involving Trace-Squared Term with Nonexotic Matter

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Received 7 October 2020; Revised 23 March 2021; Accepted 2 June 2021; Published 19 June 2021

Academic Editor: Kadri Yakut

In Einstein's relativity theory, the existence of traversable wormholes requires the involvement of exotic matter which violates the null energy condition (NEC). Our aim, in this article, is to construct wormhole solutions with the nonexotic matter. To achieve this, we choose an interesting gravitational framework of $f(\mathscr{R}, \mathscr{T})$ theory which contains a quadratic term of energy-momentum tensor $\mathscr{T}_{ij}$ trace and a well-known Starobinsky $f(\mathscr{R})$ model in its extended form. We analyze the behavior of energy constraints in the framework of $f(\mathscr{R}, \mathscr{T}) = \mathscr{R} + \alpha\mathscr{R}^2 + c\mathscr{R}^n + \lambda\mathscr{T}^2$ (where α, γ, and λ are some random constants) model for the well-proposed shape function $S(r) = r_0(r_0/r)^\varepsilon$ (where ε is a constant and r_0 is the wormhole throat). A detailed analysis of validity regions is presented for some choices of coupling parameters along with the free parameter of EoS (β, m). It is shown that, under this model, the existence of viable wormhole geometry is possible without requiring any exotic matter.

1. Introduction

"Speedy expanding nature of our cosmos in its current state" is one of the most captivating and recent explorations on the cosmological landscape. Astrophysicists are assured that some dominant dark and secret ingredient (contributing almost 76% in the cosmic matter) is speeding up this expansion and is termed as dark energy (DE). In this respect, the first indication was provided by Supernova Ia data [1, 2] that supported the accelerated expanding cosmic nature. Later, this was affirmed by the outcomes of some other astronomical experiments, namely, cosmic microwave background (CMB) radiations, WMAP, and large-scale structure [3–6]. In this regard, the ΛCDM form aided in developing the coherence of theoretical results; however, it failed to give an adequate description of the underlying nature of DE and has shown serious tension in certain observations of the cosmic expansion. During the last few decades, substantial attempts have been made to develop the nonstandard theories of gravity [7–10]. The vast majority of these models have taken the form of extended theories of gravity, where GR is assumed exactly with extensions taken at the level of the action. The formulation of different modified theories include the fundamental extension of GR, namely, $f(\mathscr{R})$ [11], and its versions like $f(\mathscr{R}, \mathscr{T})$ (where $\mathscr{T}$ represents the trace of energy-momentum tensor T_{ij}) and $f(T)$ gravity with T as the torsion scalar [12], $f(R, G)$ theory, where G is the Gauss–Bonnet term, Gauss–Bonnet gravity [13–15], and scalar-tensor theories like Brans–Dicke theory [16].

The action of $f(\mathscr{R}, \mathscr{T})$ theory is described as [17]

$$I = \frac{1}{16\pi}\int f(\mathscr{R}, \mathscr{T})\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x, \quad (1)$$

where L_m stands for matter Lagrangian and g the determinant of metric tensor g_{ij}. In literature, three possible representations of $f(\mathscr{R}, \mathscr{T})$ model are as follows:

(1) $f(\mathscr{R}, \mathscr{T}) = \mathscr{R} + \lambda\mathscr{T}$
(2) $f(\mathscr{R}, \mathscr{T}) = f_1(\mathscr{R}) + f_2(\mathscr{T})$
(3) $f(\mathscr{R}, \mathscr{T}) = f_1(\mathscr{R}) + f_2(\mathscr{R})f_3(\mathscr{T})$

First, two options represent the minimal interaction of $\mathscr{R}$ and $\mathscr{T}$, whereas the nonminimal coupling models can be of the form given in 3. The generic nonminimal models are formulated in [18], where the authors showed the possible phase transition of cosmos from the decelerating phase to the accelerating state by presenting complete cosmic evolution (including ΛCDM, phantom, as well as non-phantom epochs) within this gravitational framework. The investigation about the thermodynamical laws' validity has been presented in [19], where it was found that, due to the presence of geometric interaction, the equilibrium picture of thermodynamical laws cannot be accomplished. For an interesting model of $f(\mathcal{R},\mathcal{T})$ gravity, Alvarenga et al. [20] explored the scalar cosmological perturbations and presented the possible constraints on the standard continuity equation. In [21], Moraes et al. presented the $f(\mathcal{R},\mathcal{T}) = \mathcal{R} + \exp\mathcal{T}$ model and investigated the evolution of Hubble and deceleration parameters. They confronted their predictions with the observational Hubble data set. The quadratic curvature term with logarithmic trace model, that is, $f(\mathcal{R},\mathcal{T}) = \mathcal{R} + \lambda\mathcal{R}^2 + 2\beta\ln\mathcal{T}$ is discussed in [22], where the authors discussed the cosmic evolution depending on two equation of state (EoS) parameters and confronted their results with the Hubble telescope experimental data. Zubair and Azmat [23, 24] discussed the idea of complexity factor for nonstatic self-gravitating source exhibiting spherical/cylindrical symmetric properties and being filled with anisotropic matter contents. Other cosmic issues including bouncing models, phantom cosmology, compact stars, and gravitational instability of collapsing stars have been explored in the literature; for instance, see [22–30]. In [31], the authors introduced the generalization of GR by involving a term proportional to $\mathcal{T}_{\alpha\beta}\mathcal{T}^{\alpha\beta}$ and formulated the corresponding set of field equations. In cosmological dynamics, it is seen that after matter dominated cosmic epochs, ΛCDM model plays a significant contribution while it does not depict any noticeable role in the early times. The cosmological implications of energy-momentum squared gravity is presented in [32], where the authors discussed early and later cosmic evolution stages including accelerated expansion, and the existence or evasion of singularities.

During recent few decades, wormholes (WHs) appear as fascinating objects that provide an interconnecting path between two distinct regions of the same cosmos or different universes. The theoretical formulation of WHs was proposed in 1916 [33] followed by a nontraversable Einstein–Rosen bridge connecting two different mouths of Schwarzchild geometry [34]. In [35], Morris and Thorne introduced the wormhole metric:

$$ds^2 = e^{2\Phi(r)}dt^2 - \frac{dr^2}{1 - S(r)/r} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad (2)$$

where $\Phi(r)$ and $S(r)$ are the symbolic notations for redshift and shape function, respectively. It was shown that WHs can be traversable but it necessitates the involvement of some exotic matter (the matter which is incompatible with the null energy bound) at throat (for keeping it open). The authors have made attempts to minimize the impact of the so-called exotic matter. Bekenstem and Mtlgrom [36] worked on the possibility of traversable solutions by considering quantum effects into account. Maldacena and Qi [37] proposed $A\ dS_2$ solution by taking the impact of an external interaction between two boundaries into account and the quantum effects resulting in negative null energy and they have shown that these assumptions can explain the eternal traversable WH. In modified gravitational theories, WHs have been studied in the literature where it is seen that the need for exotic matter can be compensated [38–47]. It has been shown that solutions representing WHs can be obtained for the nonexotic matter. In 2016, M. Zubair et al. [48] explored the existence of some interesting wormhole geometries in the framework of $f(\mathcal{R},\mathcal{T})$ gravity for anisotropic, isotropic, and barotropic matter contents and they analyzed the behavior of energy constraints for these matter sources. They concluded that the wormhole solution with anisotropic fluid is realistic and stable. Further, they extended this study to noncommutative geometric background [49]. In another study [50], the existence of WHs has been analyzed within the same gravity by considering a simple and linear model defined as $f(\mathcal{R},\mathcal{T}) = \mathcal{R} + 2\lambda\mathcal{T}$ along with the matter Lagrangian given by $L_m = -\rho$ (here the ordinary matter density is represented by symbol ρ). They formulated viable shape functions corresponding to EoS: $p_l = np_r$ and $p_r = -\omega(r)\rho$ (with $\omega(r) = \text{constant}$, $\omega(r) = Br^m$). In another paper [49], by including noncommutative geometry aspects of string theory within the $f(R,T)$ framework, researchers have proposed wormhole geometries where simple linear and cubic forms of $f(R,T)$ function were considered. In the framework of $f(\mathcal{R},\mathcal{T})$ gravity, the idea of viable charged wormhole solutions has been presented by Moraes et al. [51]. They have assumed a simple linear generic model given by $f(\mathcal{R},\mathcal{T}) = \mathcal{R} + 2\mathcal{T}$ along with the ordinary matter as the total pressure of anisotropic fluid. Further, the existence of the static wormhole model is explored by utilizing different kinds of shape functions [52]. Recently, Sahoo et al. [53] have investigated the wormhole modeling by considering a specific general shape function in the quadratic $f(\mathcal{R},\mathcal{T})$ gravity.

In the present study, our main purpose is to investigate the traversable WHs existence without involving any role of exotic matter in a gravity theory based on the trace-squared matter contribution. Here, particularly, we will insert quadratic term in involving trace of the energy-momentum tensor $\mathcal{T}_{ij}$ in the Einstein Hilbert gravitational action. Therefore, our background theory will be $f(\mathcal{R},\mathcal{T})$ gravity theory with $f(\mathcal{R},\mathcal{T}) = f_1(\mathcal{R}) + f_2(\mathcal{T})$, where $f_1(R)$ is the Starobinsky model and $f_2(\mathcal{T}) = \lambda T^2$ with λ as a coupling constant. This paper is designed as follows: Section 2 presents a short introduction of the considered gravitational framework, namely, $f(\mathcal{R},\mathcal{T})$ gravity with a quadratic term of trace $\mathcal{T}$, and defines the basic mathematical background of this framework. The next section relates to the existence of a wormhole in the trace of energy-momentum tensor squared gravity. We evaluate the energy constraints and analyze the validity regions of these conditions in Section 4. The last segment summarizes the whole discussion and highlights some important conclusions.

2. Basics of $f(\mathcal{R},\mathcal{T})$ Gravitational Framework

Here, we shall define the basic mathematical structure of this gravitational framework and define the assumptions taken for this work. The metric tensor g_{ij} variation of the above action results in the following set of field equations:

$$\begin{aligned}&8\pi\mathcal{T}_{ij}-f_{\mathcal{T}}(\mathcal{R},\mathcal{T})\mathcal{T}_{ij}-f_{\mathcal{T}}(\mathcal{R},\mathcal{T})\Theta_{ij}\\&=f_{\mathcal{R}}(\mathcal{R},\mathcal{T})\mathcal{R}_{ij}-\frac{1}{2}f(\mathcal{R},\mathcal{T})g_{\mu\nu}\\&+\left(g_{ij}\Box-\nabla_i\nabla_j\right)f_{\mathcal{R}}(\mathcal{R},\mathcal{T}),\end{aligned}\tag{3}$$

where the symbolic notations ∇_α and $\Box$, respectively, refer to the covariant derivative and four-dimensional Levi-Civita covariant derivative while $f_{\mathcal{R}}(\mathcal{R},\mathcal{T})=\partial f(\mathcal{R},\mathcal{T})/\partial\mathcal{R}$, $f_{\mathcal{T}}(\mathcal{R},\mathcal{T})=\partial f(\mathcal{R},\mathcal{T})/\partial\mathcal{T}$. Here, we shall assume the ordinary matter contents defined in terms of locally anisotropic fluid distribution whose energy-momentum tensor is given by

$$T_{ij}=(\rho+p_t)U_iU_j+p_tg_{ij}+(p_r-p_t)\chi_i\chi_j,\tag{4}$$

where ρ corresponds to the matter energy density and p_r and p_t show the radial and transverse pressure components, respectively. Here, U_i and χ_i denote four-velocity and unit four-vector along the radial direction. Under the comoving relative motion, these quantities are defined as

$$\begin{aligned}U^i&=e^{-a}\delta^i_0,\\\chi^i&=e^{-b}\delta^i_1,\\U^iU_i&=1,\\\chi^i\chi_i&=-1.\end{aligned}\tag{5}$$

The term $\Theta_{\alpha\beta}$ has the following mathematical representation:

$$\Theta_{ij}=\frac{g^{\mu\nu}\delta\mathcal{T}_{\mu\nu}}{\delta g^{ij}}=-2\mathcal{T}_{ij}+g_{ij}\mathcal{L}_{(m)}-2g^{\rho\nu}\frac{\partial^2\mathcal{L}_{(m)}}{\partial g^{ij}\partial g^{\rho\nu}}.\tag{6}$$

Here, we consider the choice of $\mathcal{L}_{(m)}=-P=-((p_r+2p_t)/3)$ (total pressure) which consequently results into $\Theta_{\alpha\beta}$ given by the following form:

$$\Theta_{ij}=-2\mathcal{T}_{ij}-Pg_{ij}.\tag{7}$$

Hence, the resultant dynamical equations take the following form:

$$\begin{aligned}&8\pi\mathcal{T}_{ij}+f_{\mathcal{T}}(\mathcal{R},\mathcal{T})\mathcal{T}_{ij}+Pf_{\mathcal{T}}(\mathcal{R},\mathcal{T})\\&=f_{\mathcal{R}}(\mathcal{R},\mathcal{T})\mathcal{R}_{ij}-\frac{1}{2}f(\mathcal{R},\mathcal{T})g_{\mu\nu}\\&+\left(g_{ij}\Box-\nabla_i\nabla_j\right)f_{\mathcal{R}}(\mathcal{R},\mathcal{T}).\end{aligned}\tag{8}$$

In our discussion, we select the Lagrangian $f(\mathcal{R},\mathcal{T})$ of the following form:

$$\begin{aligned}f(\mathcal{R},\mathcal{T})&=f_1(\mathcal{R})+f_2(\mathcal{T}),\ \text{with,}\\f_1(\mathcal{R})&=\mathcal{R}+\alpha\mathcal{R}^2+\gamma\mathcal{R}^n,\\f_2(\mathcal{T})&=\lambda\mathcal{T}^2,\end{aligned}\tag{9}$$

where $n\geq3$, α and γ are random parameters, and λ is the coupling parameter. Here, $f_1(R)$ represents the R^n extension of the prominent Starobinsky model [11]. Model (9) corresponds to the quadratic term in the trace of energy-momentum tensor which has been explored under various aspects [54, 55].

3. Wormhole Existence in Gravity Involving the Squared Trace of Energy-Momentum Tensor

Using the Morris and Thorne spacetime, equation (8) takes the following form:

$$\begin{aligned}\frac{S'}{r^2}&=\frac{1}{f_{\mathcal{R}}(\mathcal{R},\mathcal{T})}\left[(8\pi+f_{\mathcal{T}}(\mathcal{R},\mathcal{T}))\rho+\frac{(p_r+2p_t)f_{\mathcal{T}}(\mathcal{R},\mathcal{T})}{3}+\frac{f(\mathcal{R},\mathcal{T})-\mathcal{R}f_{\mathcal{R}}(\mathcal{R},\mathcal{T})}{2}+\left(1-\frac{S}{r}\right)f''_{\mathcal{R}}(\mathcal{R},\mathcal{T})\right.\\&\left.-\frac{(S'r+3S-4r)}{2r^2}f'_{\mathcal{R}}(\mathcal{R},\mathcal{T}),\frac{1}{r}\left[\frac{S}{r^2}+2\Phi'\left(\frac{S}{r}-1\right)\right]=\frac{1}{f_{\mathcal{R}}(\mathcal{R},\mathcal{T})}\left[(8\pi+f_{\mathcal{T}}(\mathcal{R},\mathcal{T}))(-p_r)\right.\right.\\&+\frac{(p_r+2p_t)f_{\mathcal{T}}(\mathcal{R},\mathcal{T})}{3}+\frac{f(\mathcal{R},\mathcal{T})-\mathcal{R}f_{\mathcal{R}}(\mathcal{R},\mathcal{T})}{2}+\frac{(S'r+S-2r)}{r^2}f'_{\mathcal{R}}(\mathcal{R},\mathcal{T}),\frac{1}{2r}\left[\frac{1}{r}\left(\Phi'S+S'-\frac{S}{r}\right)\right.\\&\left.+2\left(\Phi''+(\Phi')^2\right)S-\Phi'(2-S')\right]=\frac{1}{f_{\mathcal{R}}(\mathcal{R},\mathcal{T})}(8\pi+f_{\mathcal{T}}(\mathcal{R},\mathcal{T}))(-p_t)\\&-\frac{(p_r+2p_t)f_{\mathcal{T}}(\mathcal{R},\mathcal{T})}{3}-\frac{f(\mathcal{R},\mathcal{T})-\mathcal{R}f_{\mathcal{R}}(\mathcal{R},\mathcal{T})}{2}-\left(1-\frac{S}{r}\right)f''_{\mathcal{R}}(\mathcal{R},\mathcal{T})+\frac{(S'r+5S-6r)}{2r^2}f'_{\mathcal{R}}(\mathcal{R},\mathcal{T}).\end{aligned}\tag{10}$$

In trace-squared formalism, it seems a cumbersome task to test the matter contents and the inclusion of some relations between density and pressure will be handy in such a scenario. We pick the EoS parameters of the following form [56]:

$$p_r = m\rho,\quad p_t = \beta\rho. \tag{11}$$

For this WH study, we choose redshift function as constant, that is, $\Phi'(r) = 0$ and a well-known shape function [41, 57] given by

$$S(r) = r_0\left(\frac{r_0}{r}\right)^{\varepsilon}, \tag{12}$$

where ε is a constant and r_0 is the throat of WH. This choice of shape function has been extensively studied in the literature and it holds all the necessary constraints for the existence of WH geometry. One can see that the shape function trivially satisfies the condition $S(r = r_0) = r_0$ and also the flaring-out condition $(S(r) - S'(r)r/S(r)^2) > 0$ at $r = r_0$ requires $\varepsilon > -1$. In Table 1, we show the shape functions corresponding to different values of ε. Herein, we set $\varepsilon = 0.5$.

Under the consideration of the above-defined shape function and EoS parameters, one can find the following relations for ρ, p_r, and p_t as

$$\rho = \frac{1}{\zeta_1}\left[3rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right],$$

$$p_r = \frac{1}{\zeta_1}\left[3mrr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right],$$

$$p_t = \frac{1}{\zeta_1}\left[3\beta rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right], \tag{13}$$

where $\zeta_1, \zeta_2, \zeta_3$, and ζ_4 are defined in the appendix.

4. Energy Constraints in the Trace of Energy-Momentum Tensor Squared Gravity

This section is devoted to exploring the possible conditions on the free parameters by analyzing the validity of energy constraints for the considered model. These conditions play a key role not only in GR but also in its extended gravitational frameworks for exploring the viability of proposed models. In fact, the Raychaudhuri equation plays an important role to prove the four kinds of energy constraints which have been formulated in GR [58]. These constraints are named the null energy condition (NEC), the weak energy condition (WEC), the dominant energy condition (DEC), and the strong energy condition (SEC). For the congruence of geodesics with time-like and null-like characteristics, the well-famed Raychaudhuri's equations are defined as [59]

Table 1: Shape functions corresponding to different values of ε.

ε	1	0.5	0.2	0	−0.5
$S(r)$	(r_0^2/r)	$r_0\sqrt{(r_0/r)}$	$r_0^{6/5}r^{-1/5}$	r_0	$\sqrt{r_0r}$

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} - \mathcal{R}_{ij}u^iu^j,\quad \frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} - \mathcal{R}_{ij}k^ik^j, \tag{14}$$

where σ_{ij}, ω^{ij}, and $\mathcal{R}^{ij}$ are being the shear tensor, rotation, and Ricci tensor, respectively, while u^i and k^i represent the tangent vectors for time-like and null-like geodesics in the congruence. For $\theta < 0$, it will be converging which leads to the condition $(d\theta/d\tau) < 0$. By neglecting all second-order terms and integrating the above equation, we obtain $\theta = -\tau\mathcal{R}_{ij}u^iu^j$ and $\theta = -\tau\mathcal{R}_{ij}k^ik^j$. Also, $\sigma_{ij}\sigma^{ij} \geq 0$ (shear stress is purely spatial), $\omega = 0$ (for hypersurface orthogonal congruence), the constraints take the following form:

$$\mathcal{R}_{ij}u^iu^j \geq 0,\quad \mathcal{R}_{ij}k^ik^j \geq 0. \tag{15}$$

One can rewrite the above equations as the linear combination of the energy-momentum tensor and its trace by applying the dynamical field equations as follows:

$$\left(\mathcal{T}_{ij} - \frac{\mathcal{T}g_{ij}}{2}\right)u^iu^j \geq 0,\quad \mathcal{T}_{ij}k^ik^j \geq 0. \tag{16}$$

Now, by adopting the above definition, we obtain all energy conditions for imperfect fluid in the trace of energy-momentum tensor squared gravity as

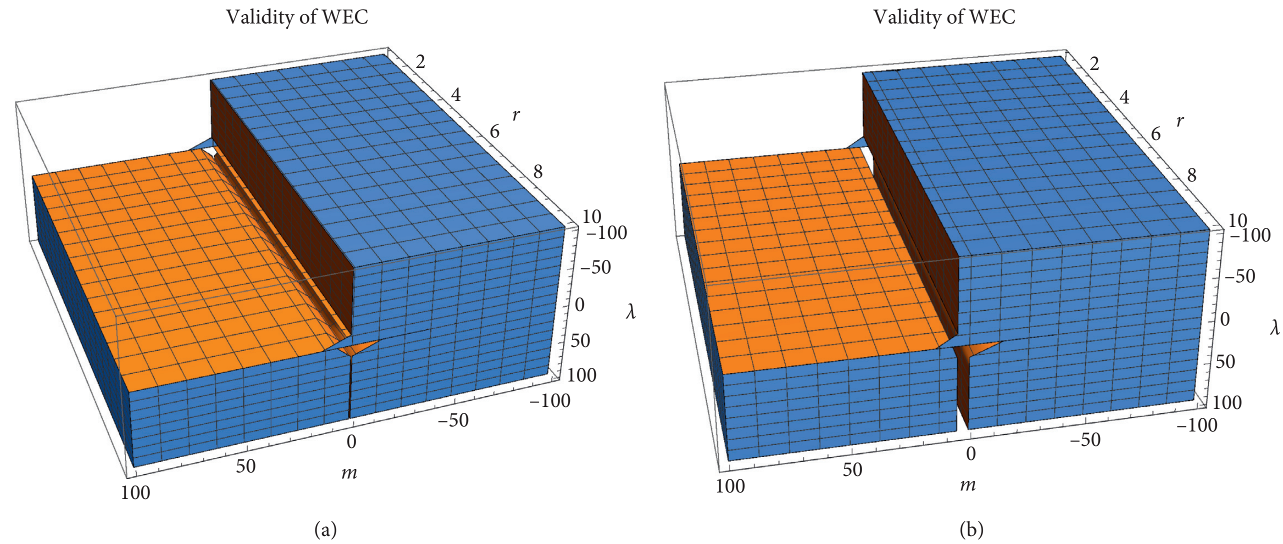


FIGURE 1: Variation of energy constraint for $f(\mathscr{R},\mathscr{T})$ model with the quadratic term in trace $\mathscr{T}$ using $\alpha > 0$, $\gamma > 0$. The left plot indicates the possible regions of WEC for $\beta > 0$, where we fix $\beta = 2$ and take the variations of m and λ. In the right plot, viability regions of WEC for $\beta < 0$ are provided, where we select $\beta = -2$.

$$\begin{aligned}
&\textbf{NEC}:\ \rho + p_r = \frac{1}{\zeta_1}\left[3(m+1)rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right] \geq 0\ \textbf{With},\\
&\rho + p_t = \frac{1}{\zeta_1}\left[3(\beta+1)rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right] \geq 0,\\
&\textbf{WEC}:\ \rho = \frac{1}{\zeta_1}\left[3rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right] \geq 0,\\
&\textbf{DEC}:\ \rho - |p_r| = \frac{1}{\zeta_1}\left[3(1-m)rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right] \geq 0\ \textbf{With},\\
&\rho - |p_t| = \frac{1}{\zeta_1}\left[3(1-\beta)rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right] \geq 0,\\
&\textbf{SEC}:\ \rho + p_r + 2p_t = \frac{1}{\zeta_1}\left[3(1+m+2\beta)rr_0\left(\frac{r_0}{r}\right)^{-2-\varepsilon}\zeta_2\left(\zeta_3 - \left(\zeta_3^2 + \frac{\zeta_4}{3r^9\zeta_2^2}\right)^{1/2}\right)\right] \geq 0.
\end{aligned} \tag{17}$$

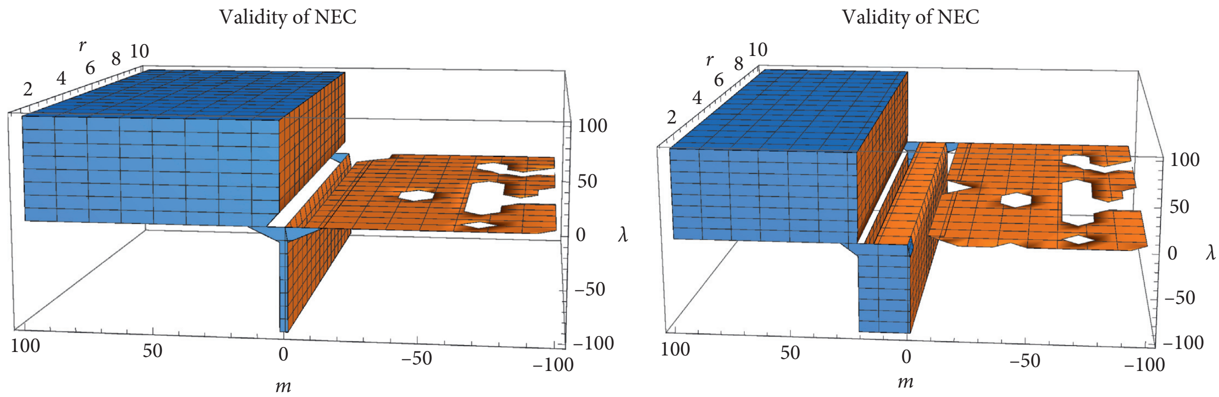
The above inequalities include five unknown parameters, namely, α, γ, m, β, and λ. In this work, we apply variation to the $f_1(\mathscr{R})$ model parameters, that is, α and γ, and find the validity regions by evaluating the feasible ranges of m, λ, and β. The viability regions for all possible cases are presented in Table 2.

(i) $\alpha > 0$ and $\gamma > 0$.

Initially, we set $\alpha > 0$ and $\gamma > 0$ to explore the validity region of WEC and NEC by taking different values of β, m, and λ. In this case, WEC is valid for $\lambda > 0$ depending on particular ranges of β and m. It is true if (i) $(\beta = 1, 0, -1, \forall m)$, (ii) $(\beta > 1, m > 1)$, here m is increasing with increasing β, and (iii) $(\beta < -1, m < -1)$, m is decreasing with decreasing β. For $\lambda < 0$, the validity of WEC necessitates (i) $(\beta > 1, \forall m < 0)$ and (ii) $(\beta \leq -1,\ m > 0$ and increases).

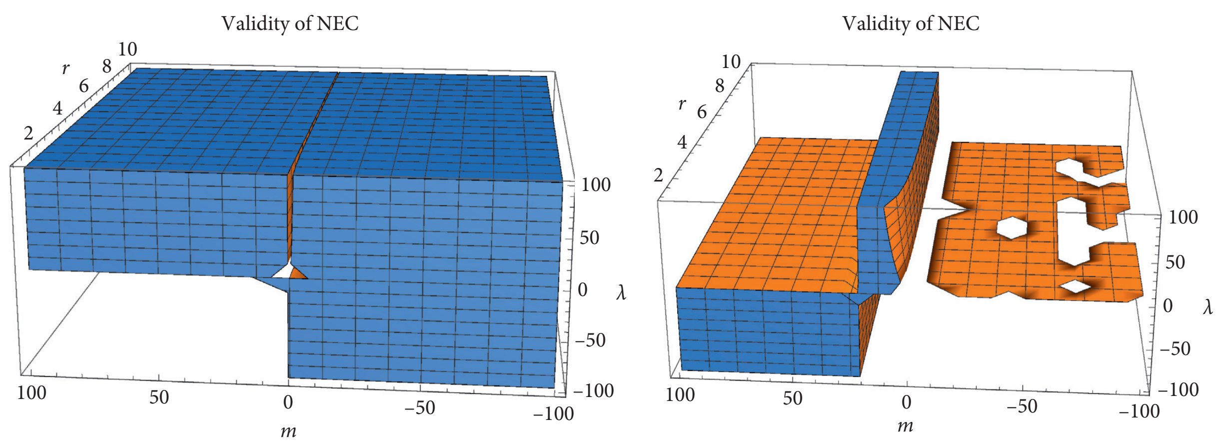
NEC-1 $(\rho + p_r \geq 0)$ is valid (i) if $\lambda > 0$ under suitable ranges $(\beta = 0$ and $\beta \geq 0$ with $m > 0)$; the value of m increases depending on the possible options of β (ii) if $\lambda < 0$ with $(\beta < 0, m > 0)$.

NEC-2 $(\rho + p_t \geq 0)$ is valid (i) if $\lambda > 0$ under suitable ranges $(\beta = 0, \forall m)$, $(\beta > 0,\ m > 0$, or $m < -2)$, and $(\beta < -1, m > 0)$ and (ii) if $\lambda < 0$ with



(a) (b)

Figure 2: For $\alpha > 0$, $\gamma > 0$. The left plot shows the feasible regions of NEC-1 $(\rho + p_r \geq 0)$ for $\beta > 0$ and we show the variation of m and λ. In the right plot, viability regions of NEC-1 $(\rho + p_r \geq 0)$ for $\beta < 0$ are presented.



(a) (b)

Figure 3: Taking $\alpha > 0$, $\gamma > 0$. The left plot shows the feasible regions of NEC-2 $(\rho + p_t \geq 0)$ for $\beta > 0$ and shows the variation of m and λ, while the right plot indicates viability regions of NEC-2 $(\rho + p_t \geq 0)$ for $\beta < 0$.

$(\beta < -1, m > 0)$. Also, in case of $\beta = -1$, we find validity $\forall m$ and $\forall \lambda$.

For the first case, we show the evolution of WEC, NEC-1, and NEC-2 in Figures 1, 2, and 3 with some particular ranges.

(ii) $\alpha > 0$ and $\gamma < 0$.

For this choice, we find that WEC is valid if $\lambda > 0$ with $(\beta = 0, -1, \forall m)$, $(\beta > 0, m < -2$ or $m > 1)$, and $(\beta < -1, m < 0)$. In case of $\lambda < 0$, validity of WEC requires $(\beta > 0, m < 0)$ and $(\beta < -1, m > 0)$. Here, the behavior of m is opposite to that of β.

NEC-1 $\rho + p_r \geq 0$ is satisfied for (i) $\lambda > 0$ and $m > 0$ with constraints $\beta \geq 0$ or $\beta \leq -1$ and (ii) $\lambda < 0$ and $m > 0$ with $\beta < -1$. In case of NEC-2 $(\rho + p_t \geq 0)$, it requires the constraints (i) $\lambda > 0$ with $(\beta = 0, \forall m)$ and $(\beta > 0, m > 0)$ and (ii) $\lambda > 0$ with $(\beta < -1, m > 0)$.

(iii) $\alpha < 0$ and $\gamma > 0$.

The validity constraint for WEC is very similar to the previous case with only the difference $\beta < -1$, $m > 0$ for $\lambda < 0$. Here, NEC-2 is met for the validity constraints, (i) $\beta = -1, \forall m$ and $\forall \lambda$, (ii) $\beta > 0$, $\lambda > 0$ with $m < -2$ or $m > 1$, and (iii) $\beta < -1, m > 0$ and $\lambda < 0$.

(iv) $\alpha < 0$ and $\gamma < 0$.

In the last case, we find that WEC develops one additional constraint $(\beta > 0$, $m < 0$ with $\lambda < -1)$ as compared to the previous cases. We found similar constraints for NEC-1 and NEC-2 as discussed in the previous case $\alpha < 0$ and $\gamma > 0$.

5. Discussion

In the current theoretical framework, it is suggested that modifications of GR provide significant intuitions towards the complicated issues of the current cosmic picture and anonymous stellar objects. In this respect, $f(\mathscr{R}, \mathscr{T})$ theory

Table 2: Validity regions of WEC and NEC in $f(\mathscr{R},\mathscr{T})$ gravity with the quadratic term in trace $\mathscr{T}$.

Variations of α and γ	WEC $(\rho \geq 0)$	NEC-1 $(\rho + p_r \geq 0)$	NEC-2 $(\rho + p_t \geq 0)$
$\alpha > 0$, $\gamma > 0$	$\beta = -1, 0, 1, \forall m$ and $\lambda > 0$ $\beta > 1$, $m > 1$ and increasing, $\lambda > 0$ or Or $m < -2$ $\beta > 1$, $m < 0$, and $\lambda < 0$ $\beta < -1$, $m < -1$, decreasing and $\lambda > 0$ $\beta < -1, m > 0$, increasing, $\lambda < 0$	$\beta = -1, 0$, $m > 0$, and $\lambda > 0$ $\beta > 0$, $m > 0$ and increasing, $\lambda > 0$ $\beta < -1$, $m > 0$ and increasing, $\lambda > 0$ $\beta < 0$, $m > 0$ and increasing, $\lambda < 0$	$\beta = 0, \forall m$, and $\lambda > 0$ $\beta > 0$, $m > 1$, or $m < -2$ And $\lambda > 0$ $\beta = -1, \forall m$, and λ $\beta < -1$, $m > 0$ Increasing, $\lambda > 0$, $\beta < -1$, $m > 0$, $\lambda < 0$
$\alpha > 0$, $\gamma < 0$	$\beta = -1, 0, \forall m$ and $\lambda > 0$ $\beta > 0$, $m < -2$ or $m > 1$ and $\lambda > 0$ or $\beta > 0$, $\forall m < 0$, $\lambda < 0$ $\beta < -1$, $m > 0$ and increasing and $\lambda < 0$ $\beta < -1$, $m < 0$, and $\lambda > 0$	$\beta = -1, 0$, $m > 0$, and $\lambda > 0$ $\beta > 0$, $m > 0$ and increasing, $\lambda > 0$ $\beta < -1$, $m > 0$, increasing and $\lambda > 0$ $\beta < -1$, $m > 0$, increasing and $\lambda < 0$	$\beta = 0, \forall m$ and $\lambda > 0$ $\beta > 0$, $m > 0$ Increasing, $\lambda > 0$ $\beta < 0$, $m > 0$, and $\lambda < 0$, Decreasing
$\alpha < 0$, $\gamma > 0$	$\beta = -1, 0, \forall m$, and $\lambda > 0$ $\beta > 0$, $m > 1$ or $m < -2$ and $\lambda > 0$ $\beta > 0$,$\forall m < 0$, $\lambda < 0$ $\beta < -1$, $m < 0$, and $\lambda > 0$ $\beta < -1$, $m > 0$, $\lambda < 0$	$\beta = -1, 0$, $m > 0$, and $\lambda > 0$ $\beta > 0$, $m > 0$ increasing and $\lambda > 0$ $\beta < -1$, $m > 0$, increasing and $\lambda < 0$ $\beta < -1$, $m > 0$ increasing and $\lambda > 0$	$\beta = 0, \forall m$ and $\lambda > 0$ $\beta > 0$, $m < -2$ or $m > 1$ And $\lambda > 0$ $\beta = -1, \forall m$ and λ $\beta < -1$, $m > 0$, and $\lambda < 0$
$\alpha < 0$, $\gamma < 0$	$\beta = -1, 0, \forall m$, and $\lambda > 0$ $\beta > 0$, $m > 0$ and increasing, $\lambda > 0$ or $\beta > 0$, $m < 0$, $\lambda < -1$ and decreasing $\beta < -1$, $m < 0$ and decreasing, $\lambda > 0$	$\beta = -1, 0$, $m > 0$, and $\lambda > 0$ $\beta > 0$, $m > 0$ increasing and $\lambda > 0$ $\beta < -1$, $m > 0$ increasing and $\lambda > 0$ $\beta < -1$, $m > 0$, increasing and $\lambda < 0$	$\beta = 0, \forall m$ and $\lambda > 0$ $\beta > 0$, $m < -2$, or $m > 1$ And $\lambda > 0$ $\beta = -1, \forall m$ and λ $\beta < -1$, $m > 0$, and $\lambda < 0$

appears as one of the fascinating and strongest alternatives, which involves the concept of curvature-matter nonminimal interaction and it also provides the corrections with higher-order correction terms. In this manuscript, we have selected the $f(\mathscr{R},\mathscr{T}) = \mathscr{R} + \alpha\mathscr{R}^2 + \gamma\mathscr{R}^n + \lambda\mathscr{T}^2$ generic model.

In 1988, Morris and Thorne [35] proposed the idea of traveling through WHs. They discussed the static spherically symmetric geometry of WHs and found that the involvement of the exotic kind of matter is fundamental to alleviate the traversable nature of such objects. In GR, one needs to include the exotic source of energy to explain the WH solutions, there can be different possibilities like cosmological constant or any other equation of state parameter can be taken into account. However, in the modified theories, one can develop such objects by excluding the impact of exotic matter. Harko et al. [60] considered the nonexotic matter to formulate static spherically symmetric wormholes in simple $F(\mathscr{R})$ theory. Here, we are interested in exploring the WH solutions in $f(\mathscr{R},\mathscr{T})$ gravity with the quadratic term in trace of the energy-momentum tensor and present some constraints for the existence and viability of such solutions.

On theoretical background, it is argued that highly compact objects carry unequal pressures, that is, such objects involve anisotropic pressure. Anisotropic matter distribution is more generic as compared to barotropic/isotropic one and it is suggested that such matter would be more useful to examine the WH existence. There are different techniques to evaluate the existence of WH solutions, one such scheme is to select the shape function and check its feasibility for the considered modified theory. In this manuscript, we have opted for the known shape function given by $S(r) = r_0(r_0/r)^{\varepsilon}$ which satisfies all the imperative constraints for the existence of WHs along with anisotropic fluid distribution. With this choice of $S(r)$, we have checked the validity of energy constraints. It is found that WEC $(\rho \geq 0)$, NEC-1 $(\rho + p_r \geq 0)$, and NEC-2 $(\rho + p_t \geq 0)$ depend on five arbitrary parameters, namely, α, γ, m, β, and λ. In this procedure, we have fixed α and γ and observed the feasible validity regions by changing other parameters. All the obtained results are summarized in Table 2.

There are many papers available on this subject. For example, in [50], the authors discussed the existence of exact traversable wormholes in $f(R,T)$ theory by taking its linear form $f(R,T) = R + 2\lambda T$ and different EoS parameters into account. They showed that their obtained solutions violate the NEC. Likewise, in another paper [61], a linear model $f(R,T) = R + \lambda T$ along with radial EoS parameter is considered and exact solutions are obtained for this simple case. It was concluded that, for some certain ranges of parameters, the obtained wormhole solution satisfies the null energy condition. In [48], the authors investigated the existence of static spherically symmetric wormhole solutions in $f(R,T)$ theory by taking the Starobinsky model with $n = 3$ and the linear form $f(T) = \lambda T$. For this study, they considered the power law form of shape function along with anisotropic, barotropic, and isotropic fluids and analyzed the energy constraints for exploring the possibility of wormholes' existence without requiring any type of exotic matter. In case of anisotropic fluid, they found that, in few regions of spacetime (for a specific small range of r and α), wormholes' existence is possible without requiring any type of exotic matter.

In this respect, Mishra et al. [62] analyzed the nature of SEC and WEC in the context of $f(R,T) = R + \Lambda T$ gravity with two different types of shape functions. For suitable choices of free parameters, they obtained wormhole solutions for which both NEC and SEC are valid, while DEC is also compatible in terms of p_l and only DEC is violated for radial pressure p_r in both the models. Consequently, it was concluded that the existence of traversable wormhole solutions for the considered configuration can be ensured without requiring exotic matter. In another paper [63], the authors discussed the existence of wormholes in the $f(R,T)$ framework by taking the $f(R,T) = R + \gamma e^{\chi T}$ model along with EoS parameters and shape function ansatz into account. It was shown that exponential $f(R,T)$ gravity can ensure the existence of wormhole solutions satisfying all the energy bounds. In [55], Moraes and Sahoo used an interesting gravitational framework involving trace-squared gravity defined by $f(R,T) = R + \alpha T + \beta T^2$ for wormhole modeling. They concluded that energy constraints are valid for a wide range of values of r and free parameters in the absence of exotic matter. In another study [52], the authors focused on the existence of wormhole solutions for $f(R,T) = R + 2\lambda T$, where EoS parameters are used for radial and tangential pressures. They concluded that their results are in good agreement with the previous works on this subject. In a recent paper [64], the authors discussed the possibility of traversable wormhole geometry in the traceless version of the $f(R,T) = R + 2\lambda T$ theory and used $p_r = \omega\rho$ EoS. They have shown that the obtained wormhole geometry requires only a small amount of exotic matter and hence is compatible with the causality.

In all these papers, either linear terms of curvature or linear terms of energy-momentum tensor trace are considered. The present paper can be considered as a generalization of all these works in the sense that here we are considering a general $f(R,T) = R + \alpha R^2 + \gamma R^n + \lambda T^2$ model which involves not only higher-order curvature terms but also the quadratic term in the trace of the energy-momentum tensor. In our case, we have found the validity regions for both WEC and NEC with a wide range of r and free parameters, hence ensuring the existence of traversable wormholes in this gravity without requiring any exotic matter. It can be concluded that, due to the involvement of higher-order curvature terms and quadratic term of trace of T_{ij}, the validity of energy condition is possible in this gravity.

Appendix

$$
\begin{aligned}
\zeta_1 &= 2\varepsilon(-3 + 11m - 2\beta)(-1 + m + 2\beta)\lambda, \\
\zeta_2 &= -\frac{2\varepsilon r_0 (r_0/r)^{\varepsilon}}{r^3} + \frac{8(\varepsilon r_0)^2 \alpha (r_0/r)^{2\varepsilon}}{r^6} + 2^n n\left(-\frac{\varepsilon r_0 (r_0/r)^{\varepsilon}}{r^3}\right)^n \gamma, \\
\zeta_3 &= -\frac{8m\pi}{1 - \left(4\varepsilon r_0 (r_0/r)^{\varepsilon}\alpha/r^3\right) + 2^{-1+n} n\left(-\left(\varepsilon r_0 (r_0/r)^{\varepsilon}/r^3\right)\right)^{-1+n}\gamma}, \\
\zeta_4 &= 4(-3 + 11m - 2\beta)(-1 + m + 2\beta)\left(\frac{2\varepsilon^2 r_0^3 (r_0/r)^{3\varepsilon}\left(r^3 + 8\varepsilon(3+\varepsilon)r\alpha + 2\varepsilon\left(-8 + 5\varepsilon + 2\varepsilon^2\right)r_0 (r_0/r)^{\varepsilon}\alpha\right)}{r^3} - 2^n r^6\right. \\
&\quad \left. \cdot \left(\frac{-\varepsilon r_0 (r_0/r)^{\varepsilon}}{r^3}\right)^{1+n}\left(-\varepsilon r_0\left(\frac{r_0}{r}\right)^{\varepsilon} + (3+\varepsilon)n^2\left(2r + (-1+\varepsilon)r_0\left(\frac{r_0}{r}\right)^{\varepsilon}\right) - n\left(2(3+\varepsilon)r + \left(-2 + \varepsilon + \varepsilon^2\right)r_0\left(\frac{r_0}{r}\right)^{\varepsilon}\right)\right)\gamma\right)\lambda,
\end{aligned}
\tag{A.1}
$$

where prime denotes the derivative with respect to "r" coordinate.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

M. Zubair thanks the Higher Education Commission, Islamabad, Pakistan, for its financial support under the NRPU project with Grant no. 5329/Federal/NRPU/ R&D/HEC/ *R&D/HEC*/2016. Saira Waheed thanks Prince Mohammad Bin Fahd University, Al Khobar 31952, Saudi Arabia, for financial support.

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