AN INVERSE PROBLEM FOR A SECOND-ORDER DIFFERENTIAL EQUATION IN A BANACH SPACE

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We consider the problem of determining the unknown term in the right-hand side of a second-order differential equation with unbounded operator generating a cosine operator function from the overspecified boundary data. We obtain necessary and sufficient conditions of the unique solvability of this problem in terms of location of the spectrum of the unbounded operator and properties of its resolvent.

1. Introduction

In a Banach space E we consider the differential equation

$$\frac{d^2v}{dt^2} = Av + f(t) + p, \quad 0 \le t \le t_1. \tag{1.1}$$

Here A is a linear unbounded operator with the domain $D(A)$, $f(t)$ is a function continuous on the segment $[0, t_1]$ with the values in the space E, and p is an unknown parameter belonging to E. By the solution of the differential equation (1.1) we mean a function twice continuously differentiable on $[0, t_1]$ with the values from $D(A)$ satisfying (1.1). For (1.1) we put the boundary value conditions

$$v(0) = v_0, \qquad v'(0) = \dot{v}_0, \tag{1.2}$$

$$v(t_1) = v_1. \tag{1.3}$$

The problem is to find a pair $(v(t), p)$ which satisfies the differential equation (1.1) and the boundary value conditions (1.2), (1.3).

The inverse problems of such a type were studied by various authors; the bibliography may be found in [5]. Such a problem for a second-order differential equation was considered in [4] in the case of a differential equation with a selfadjoint operator in a Hilbert space and in [5] in the assumption that the cosine operator function generated by the operator A is small in the norm.

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In this paper, we obtain necessary and sufficient conditions for the unique solvability of the problem (1.1)–(1.3) based on the only assumption that the operator A generates a cosine operator function. We extend on the second-order differential equations the results obtained in [2] for equations of the first order. We use here results and method of the paper [1] devoted to the spectral properties of cosine operator functions.

We assume that the operator A is a generator of a cosine operator function $C(t)$. Such operator is closed, its domain $D(A)$ is dense in E, and the resolvent set of A is nonempty (see [3]). In the sequel we denote by $R(\lambda;A)$ the resolvent $(\lambda I - A)^{-1}$ of the operator A. We also assume that the other data of the problem satisfy the conditions

(a) $v_0, v_1 \in D(A)$;

(b) $\dot{v} \in E_0$, where E_0 is the subspace of all vectors u such that the function $C(t)u$ is continuously differentiable;

(c) $f(t) = f_1(t) + f_2(t)$, where $f_1(t)$ is a function continuously differentiable in $[0,t_1]$, $f_2(t) \in D(A)$, $0 \le t \le t_1$, and $Af_2(t)$ is continuous in $[0,t_1]$.

The problem (1.1)–(1.3) is called *well posed* if for any data v_0, $\dot{v}_0$, v_1, $f(t)$ satisfying these conditions, it has a unique solution. One can show (see, e.g., [5]) that the unique solvability of the problem (1.1)–(1.3) implies the estimates for the solution $(v(t),p)$ in the corresponding norm.

The numbers

$$\mu_k = -\frac{4\pi^2 k^2}{t_1^2}, \quad k = 1,2,\ldots, \tag{1.4}$$

are called *the characteristic numbers* of the problem (1.1)–(1.3).

Theorem 1.1. *The problem (1.1)–(1.3) is well posed if and only if every characteristic number μ_k, $k = 1,2,\ldots$, is a regular point of the operator A and for any $x \in E$ the series $\sum_{k=1}^{\infty} R(\mu_k, A)x$, $\sum_{k=1}^{\infty} AR^2(\mu_k,A)x$ are Cesáro summable.*

Recall that a series $\sum_{n=1}^{\infty} a_n$, $a_n \in E$, is said to be Cesáro summable if there exists the limit

$$(C-1)\sum_{n=1}^{\infty} a_n := \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N-1} \sum_{k=1}^{n} a_k \tag{1.5}$$

and a double series $\sum_{n=-\infty}^{\infty} a_n$ is said to be Cesáro summable if there exists the limit

$$(C-1)\sum_{n=-\infty}^{\infty} a_n := \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=-n}^{n} a_k. \tag{1.6}$$

Proof. The conditions on the data v_0, $\dot{v}_0$, $f(t)$ yield (see [3, pages 35-36]) that the Cauchy problem (1.1)-(1.2) has a unique solution given by the formula

$$v(t) = C(t)v_0 + S(t)\dot{v}_0 + \int_0^t S(t-s)f(s)ds + \int_0^t S(t-s)p\,ds, \quad 0 \le t \le t_1, \tag{1.7}$$

where $C(t)$ is the operator cosine function generated by the operator A and $S(t)$ is the associated sine function. Using the boundary condition (1.3) we conclude that the problem

(1.1)–(1.3) is equivalent to the operator equation

$$Bp = w, \tag{1.8}$$

where

$$Bp = \int_0^{t_1} S(t_1 - s)p\,ds, \quad p \in E, \tag{1.9}$$

$$w = v_1 - C(t_1)v_0 - S(t_1)\dot{v}_0 - \int_0^{t_1} S(t_1 - s)f(s)ds. \tag{1.10}$$

The problem (1.1)–(1.3) is well posed if and only if (1.8) has a unique solution for any $w \in D(A)$. The latter is valid if and only if the operator B defined in (1.9) has an inverse defined on all $D(A)$.

At first we derive some relations with the operators B and $\mu_k I - A$. Set $\rho_k = 2\pi ki/t_1$, $k = 0, \pm1, \pm2, \ldots$. We prove that

$$Bx = (\mu_k I - A)\int_0^{t_1}\int_0^u (s-u)S(s)x\,ds\cosh(\rho_k u)\,du, \quad x \in E,\ k = 1, 2, \ldots. \tag{1.11}$$

For $x \in D(A)$ using the equalities $AS(s)x = S''(s)x$, $S'(s)x = C(s)x$ and the fact that A is closed we have

$$\begin{aligned}
A&\int_0^{t_1}\int_0^u (s-u)S(s)x\,ds\cosh(\rho_k u)\,du \\
&= \int_0^{t_1}\int_0^u (s-u)S''(s)x\,ds\cosh(\rho_k u)\,du \\
&= \int_0^{t_1}\left[(s-u)C(s)x|_0^u - \int_0^u S'(s)x\,ds\right]\cosh(\rho_k u)\,du \\
&= \int_0^{t_1}[ux - S(u)x]\cosh(\rho_k u)\,du
\end{aligned} \tag{1.12}$$

and hence

$$A\int_0^{t_1}\int_0^u (s-u)S(s)x\,ds\cosh(\rho_k u)\,du = -\int_0^{t_1} S(u)x\cosh(\rho_k u)\,du. \tag{1.13}$$

Since A is closed and $D(A)$ is dense in E, the last equality is valid for any $x \in E$. Next we have

$$\begin{aligned}
\mu_k&\int_0^{t_1}\int_0^u (s-u)S(s)x\,ds\cosh(\rho_k u)\,du \\
&= \int_0^{t_1}\int_0^u (s-u)S(s)x\,ds\big(\cosh(\rho_k u)\big)''du \\
&= \int_0^u (s-u)S(s)x\,ds\rho_k\sinh(\rho_k u)|_0^{t_1} + \int_0^{t_1}\int_0^u S(s)x\,ds\big(\cosh(\rho_k u)\big)'du \\
&= \int_0^u S(s)x\,ds\cosh(\rho_k u)\,du|_0^{t_1} - \int_0^{t_1} S(u)x\cosh(\rho_k u)\,du
\end{aligned} \tag{1.14}$$

which implies

$$\mu_k \int_0^{t_1}\int_0^{u}(s-u)S(s)x\,ds\cosh(\rho_k u)\,du = \int_0^{t_1} S(s)x\,ds - \int_0^{t_1} S(u)x\cosh(\rho_k u)\,du. \tag{1.15}$$

Subtracting (1.13) from (1.15), we obtain (1.11). Moreover, from the formulas (1.11), (1.13), we obtain

$$ABx = -(\mu_k I - A)\int_0^{t_1} S(u)x\cosh(\rho_k u)\,du. \tag{1.16}$$

Next we check the equality

$$\int_0^{t_1} S(u)x\cosh(\rho_k u)\,du = (\mu_k I - A)\frac{1}{2}\int_0^{t_1}\int_0^{u}(t_1-s)S(s)x\,ds\cosh(\rho_k u)\,du, \quad k=1,2,\ldots, \tag{1.17}$$

for any $x\in E$. For $x\in D(A)$ we have

$$\begin{aligned} A\frac{1}{2}&\int_0^{t_1}\int_0^{u}(t_1-s)S(s)x\,ds\cosh(\rho_k u)\,du\\ &= \frac{1}{2}\int_0^{t_1}\int_0^{u}(t_1-s)S''(s)x\,ds\cosh(\rho_k u)\,du\\ &= \frac{1}{2}\int_0^{t_1}\Big[(t_1-s)C(s)x|_0^u + \int_0^{u} S'(s)x\,ds\Big]\cosh(\rho_k u)\,du \end{aligned} \tag{1.18}$$

and therefore

$$A\frac{1}{2}\int_0^{t_1}\int_0^{u}(t_1-s)S(s)x\,ds\cosh(\rho_k u)\,du = \frac{1}{2}\int_0^{t_1}[(t_1-u)C(u)x+S(u)x]\cosh(\rho_k u)\,du. \tag{1.19}$$

Since A is closed and $D(A)$ is dense in E, the last equality is valid for any $x\in E$. Next we have

$$\begin{aligned} \mu_k\frac{1}{2}&\int_0^{t_1}\int_0^{u}(t_1-s)S(s)x\,ds\cosh(\rho_k u)\,du\\ &= \frac{1}{2}\int_0^{t_1}\int_0^{u}(t_1-s)S(s)x\,ds\big(\cosh(\rho_k u)\big)''\,du\\ &= \frac{1}{2}\Big(\int_0^{u}(t_1-s)S(s)x\sinh(\rho_k u)|_0^{t_1} - \int_0^{t_1}(t_1-u)S(u)x\big(\cosh(\rho_k u)\big)'\,du\Big)\\ &= \frac{1}{2}\Big[-(t_1-u)S(u)x\cosh(\rho_k u)|_0^{t_1} + \int_0^{t_1}((t_1-u)C(u)x - S(u)x)\cosh(\rho_k u)\,du\Big] \end{aligned} \tag{1.20}$$

which implies

$$\mu_k\frac{1}{2}\int_0^{t_1}\int_0^{u}(t_1-s)S(s)x\,ds\cosh(\rho_k u)\,du = \frac{1}{2}\int_0^{t_1}((t_1-u)C(u)x - S(u)x)\cosh(\rho_k u)\,du. \tag{1.21}$$

Subtracting (1.19) from (1.21), we obtain (1.17). Moreover, from the formulas (1.16), (1.17), we obtain

$$ABx = -(\mu_k I - A)^2 \frac{1}{2} \int_0^{t_1} \int_0^{u} (t_1 - s) S(s) x \, ds \cosh(\rho_k u) \, du, \quad x \in E, \ k = 1, 2, \dots. \tag{1.22}$$

Assume that the operator B has the inverse B^{-1} with $D(B^{-1}) = D(A)$. Let λ_0 be a regular point of the operator A. Then the operator $B_0 = B(\lambda_0 I - A)^{-1}$ is bounded. From the relations (1.11) we conclude that the characteristic numbers μ_k, $k = 1, 2, \dots$, are the regular points of the operator A. Moreover, by virtue of (1.11), (1.13), we obtain

$$R(\mu_k; A)x = \int_0^{t_1} \cosh \rho_k u B_0 \left(\lambda_0 \int_0^{u} (s - u) S(s) x \, ds - S(u) x \right) du. \tag{1.23}$$

This means that

$$R(\mu_k; A)x = \int_0^{t_1} \varphi(u) \cosh(\rho_k u) \, du, \quad k = 1, 2, \dots, \tag{1.24}$$

where

$$\varphi(u) = B_0 \left(\lambda_0 \int_0^{u} (s - u) S(s) x \, ds - S(u) x \right) \tag{1.25}$$

is a continuous function. Consider the elements

$$z_k = \int_0^{t_1} \varphi(u) \cosh(\rho_k u) \, du, \quad k = 0, \pm 1, \pm 2, \dots. \tag{1.26}$$

By the vector version of Fejér's theorem, there exists the limit

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=-n}^{n} \int_0^{t_1} \varphi_k(u) e^{\rho_k u} \, du. \tag{1.27}$$

Using the relations

$$\frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=-n}^{n} \int_0^{t_1} \varphi_k(u) e^{\rho_k u} \, du = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=-n}^{n} z_k = z_0 + \frac{2}{N} \sum_{n=0}^{N-1} \sum_{k=-n}^{n} z_k \tag{1.28}$$

and (1.24), we conclude that the series $\sum_{k=1}^{\infty} R(\mu_k; A)x$ is Cesáro summable for any $x \in E$.

Next, by virtue of the relations (1.22) and (1.19), we obtain

$$AR^2(\mu_k;A)x = \int_0^{t_1} \psi(u)\cosh(\rho_k u)\,du, \quad x \in E,\ k = 1,2,\dots, \tag{1.29}$$

where

$$\psi(u) = \frac{1}{2}B_0\left(-\lambda_0\int_0^u (t_1 - s)S(s)x\,ds + (t_1 - u)C(u)x + S(U)x\right)\cosh(\rho_k u)\,du \tag{1.30}$$

is a continuous function. From here in the same way as above we conclude that the series $\sum_{k=1}^{\infty} AR^2(\mu_k;A)x$ is Cesáro summable for any $x \in E$.

Assume that every characteristic number μ_k, $k = 1,2,\dots$, is a regular point of the operator A and for any $x \in E$ the series $\sum_{k=1}^{\infty} R(\mu_k,A)x$, $\sum_{k=1}^{\infty} AR^2(\mu_k,A)x$ are Cesáro summable. Following the method of [1], define on the subspace $D(A)$ the operators Q, P via the relations

$$\begin{aligned} Qy &= 2(C-1)\sum_{k=1}^{\infty} R(\mu_k,A)Ay, \quad y \in D(A), \\ Py &= 4(C-1)\sum_{k=1}^{\infty} AR^2(\mu_k,A)Ay, \quad y \in D(A). \end{aligned} \tag{1.31}$$

Define the operators Q_k, P_k via the relations

$$\begin{aligned} Q_k x &= \frac{1}{t_1}\int_0^{t_1} S(s)x\cosh(\rho_k s)\,ds, \quad k = 0,\pm 1,\pm 2,\dots,\ x \in E, \\ P_k x &= \frac{1}{t_1}\int_0^{t_1} \left[(t_1 - s)C(s)x + S(s)x\right]\cosh(\rho_k s)\,ds, \quad k = 0,\pm 1,\pm 2,\dots,\ x \in E. \end{aligned} \tag{1.32}$$

By virtue of (1.16) we obtain

$$\frac{1}{t_1}R(\mu_k,A)ABx = -Q_k x, \quad k = 1,2,\dots,\ x \in E, \tag{1.33}$$

and using (1.22) and (1.19) we get

$$\frac{1}{t_1}AR^2(\mu_k,A)ABx = -\frac{1}{2}P_k x, \quad k = 1,2,\dots,\ x \in E. \tag{1.34}$$

By the vector version of Fejér's theorem, we have

$$(C-1)\sum_{k=-\infty}^{\infty} Q_k x = \frac{1}{2}S(t_1)x, \tag{1.35}$$

$$(C-1)\sum_{k=-\infty}^{\infty} P_k x = \frac{1}{2}\left[S(t_1)x + t_1 x\right]. \tag{1.36}$$

Using the relations $Q_0 = (1/t_1)B$ and (1.33) we obtain

$$\begin{aligned}(C-1)\sum_{k=-\infty}^{\infty} Q_k x &= Q_0 x + 2(C-1)\sum_{k=1}^{\infty} Q_k x \\ &= \frac{1}{t_1}Bx - \frac{2}{t_1}\sum_{k=1}^{\infty} R(\mu_k;A)ABx \\ &= \frac{1}{t_1}Bx - \frac{1}{t_1}QBx\end{aligned} \tag{1.37}$$

and by virtue of (1.35) we conclude that

$$\frac{1}{t_1}Bx - \frac{1}{t_1}QBx = \frac{1}{2}S(t_1)x. \tag{1.38}$$

Next we have

$$\begin{aligned}P_0 x &= \frac{1}{t_1}Bx + \frac{1}{t_1}\int_0^{t_1}(t_1 - s)S'(s)x\,ds \\ &= \frac{1}{t_1}Bx + \frac{1}{t_1}\left[(t_1 - s)S(s)x|_0^{t_1} + \int_0^{t_1} S(s)x\,ds\right] = \frac{2}{t_1}Bx\end{aligned} \tag{1.39}$$

for $x \in D(A)$ and since the operators P_0, B are bounded, we get $P_0 x = (2/t_1)Bx$ for any $x \in E$. Thus by virtue of (1.34) we obtain

$$\begin{aligned}(C-1)\sum_{k=-\infty}^{\infty} P_k x &= P_0 x + 2(C-1)\sum_{k=1}^{\infty} P_k x \\ &= \frac{2}{t_1}Bx - \frac{4}{t_1}\sum_{k=1}^{\infty} AR^2(\mu_k;A)ABx \\ &= \frac{2}{t_1}Bx - \frac{1}{t_1}PBx\end{aligned} \tag{1.40}$$

and by virtue of (1.36) we conclude that

$$\frac{2}{t_1}Bx - \frac{1}{t_1}PBx = \frac{1}{2}S(t_1)x + \frac{1}{2}t_1 x. \tag{1.41}$$

Subtracting (1.38) from (1.41) we obtain

$$\frac{1}{t_1}Bx + \frac{1}{t_1}QBx - \frac{1}{t_1}PBx = \frac{1}{2}t_1 x \tag{1.42}$$

which implies $SBx = x$, $x \in E$, with $S = (2/t_1^2)(I + Q - P)$. The operator S is defined on $D(A)$ and since S and B commute on $D(A)$, we conclude that $BSy = y$, $y \in D(A)$, and thus $S = B^{-1}$. □

Theorem 1.2. *For the problem (1.1)–(1.3) to be well-posed it is necessary, and in the case when E is a Hilbert space is sufficient, that every characteristic number μ_k, $k = 1,2,\ldots$, be a regular point of the operator A and* $\sup_{k\geq 1}\|kR(\mu_k,A)\| < \infty$.

Proof. Assume that the problem (1.1)–(1.3) is well-posed. Then as was shown above every characteristic number μ_k, $k = 1,2,\ldots$, is a regular point of the operator A; moreover, the resolvent $R(\mu_k;A)$ in these points satisfies the relations (1.23). From these relations, integrating by parts, we obtain

$$\rho_k R(\mu_k;A)x = B_0 \int_0^{t_1} \left[\lambda_0 \int_0^u S(s)x\,ds + C(u)x\right] \sinh(\rho_k u)\,du. \tag{1.43}$$

From here using the estimates (see [3])

$$||C(u)|| \le M_1, \quad ||S(u)|| \le M_2, \quad 0 \le u \le t_1, \tag{1.44}$$

we obtain $\sup_{k\ge 1} \|\rho_k R(\mu_k,A)\| < \infty$ which means $\sup_{k\ge 1} \|kR(\mu_k,A)\| < \infty$.

Assume that E is a Hilbert space and every characteristic number μ_k, $k = 1,2,\ldots$, is a regular point of the operator A and

$$\sup_{k\ge 1} ||kR(\mu_k,A)|| < \infty. \tag{1.45}$$

As was shown in [1], condition (1.45) implies that the sequences of the partial Cesáro sums

$$R_N x = \frac{1}{N}\sum_{n=1}^{N-1}\sum_{k=1}^{n} R(\mu_k;A)x, \qquad V_N x = \frac{1}{N}\sum_{n=1}^{N-1}\sum_{k=1}^{n} AR^2(\mu_k;A)x \tag{1.46}$$

are bounded for any $x \in E$. Hence in order to prove that the series $\sum_{k=1}^{\infty} R(\mu_k;A)x$, $\sum_{k=1}^{\infty} AR^2(\mu_k;A)x$ are Cesáro summable for any $x \in E$, it is enough to check this assertion for the elements from the dense subspace $D(A)$. Let λ_0 be a regular point of the operator A with $\operatorname{Re}\lambda_0 > 0$. For every z from $D(A)$ we have $z = R(\lambda_0;A)y$ with some $y \in E$. Using the resolvent identity we obtain

$$R(\mu_k;A)z = R(\mu_k;A)R(\lambda_0;A)y = \frac{1}{\mu_k - \lambda_0}\left(R(\lambda_0;A) - R(\mu_k;A)\right)y. \tag{1.47}$$

By virtue of (1.45) we conclude that

$$||R(\mu_k;A)z|| \le \frac{1}{|\mu_k - \lambda_0|}\left(||R(\lambda_0;A)|| + ||R(\mu_k;A)||\right)\|y\| \le \frac{C_1}{|\mu_k - \lambda_0|}\|y\| \tag{1.48}$$

with some constant C_1 not depending on k. Hence it follows that for any $z \in D(A)$ the series $\sum_{k=1}^{\infty} R(\mu_k;A)z$ converges and therefore is Cesáro summable.

Next we have

$$AR^2(\mu_k;A) = -R(\mu_k;A) + \mu_k R^2(\mu_k;A) \tag{1.49}$$

and thus it remains to show that the series $\sum_{k=1}^{\infty} \mu_k R^2(\mu_k;A)z$ is Cesáro summable for any $z \in D(A)$. Using the resolvent identity we get

$$\mu_k R^2(\mu_k;A)z = \mu_k R^2(\mu_k;A)R(\lambda_0;A)y$$
$$= \frac{1}{\mu_k - \lambda_0}\left(\frac{\mu_k}{\mu_k - \lambda_0}\big(R(\lambda_0;A) - R(\mu_k;A)\big) - \mu_k R^2(\mu_k;A)\right)y. \tag{1.50}$$

Hence it follows that

$$||\mu_k R^2(\mu_k;A)z||$$
$$\leq \frac{1}{|\mu_k - \lambda_0|}\left[\left|\frac{1}{\mu_k}(\mu_k - \lambda_0)\right|\big(||R(\lambda_0;A)|| + ||R(\mu_k;A)||\big) + ||\rho_k R(\mu_k;A)||^2\right]\|y\|. \tag{1.51}$$

By virtue of (1.45) we obtain

$$||\mu_k R^2(\mu_k;A)z|| \leq \frac{C_2}{|\mu_k - \lambda_0|}\|y\| \tag{1.52}$$

with some constant C_2 not depending on k. Hence it follows that for any $z \in D(A)$ the series $\sum_{k=1}^{\infty} \mu_k R^2(\mu_k;A)z$ converges and therefore is Cesáro summable. □

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